

Transmission characteristics of acoustic amplifier in thermoacoustic engine

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Abstract

Thermoacoustic engines are promising in practical applications for the merits of simple configuration, reliable operation and environmentally friendly working gas. An acoustic amplifier can increase the output pressure amplitude of a thermoacoustic engine (TE) and improve the matching between the engine and its load. In order to make full use of an acoustic amplifier, the transmission characteristics are studied based on linear thermoacoustic theory. Computational and experimental results show that the amplifying ability of an acoustic amplifier is mainly determined by its geometry parameters and output resistance impedance. The amplifying ability of an acoustic amplifier with appropriate length and diameter reaches its maximum when the output resistance impedance is infinite. It is also shown that the acoustic amplifier consumes an amount of acoustic power when amplifying pressure amplitude and the acoustic power consumption increases with amplifying ratio. Furthermore, a novel cascade acoustic amplifier is proposed, which has a much stronger amplifying ability with reduced acoustic power consumption. In experiments, a two-stage cascade acoustic amplifier amplifies the pressure ratio from 1.177 to 1.62 and produces a pressure amplitude of 0.547 MPa with nitrogen of 2.20 MPa as working gas. Good agreements are obtained between the theoretical analysis and experimental results. This research is instructive for comprehensively understanding the mechanism and making full use of the acoustic amplifier.

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1. Introduction

A thermoacoustic engine (TE) converts thermal energy into acoustic power without any mechanical moving parts. It has a simple configuration, runs reliably and is environmentally friendly [1,2]. A pulse tube cooler (PTC) eliminates mechanical moving parts at low temperature [3]. A combination of the two has shown great potential in natural gas liquefaction, microcircuit cooling and near ambient temperature refrigeration [4].

In order to improve the performance of a thermoacoustically driven refrigeration system, great efforts should be put on optimizing the match between the TE and the PTC. The effect of frequency match on performance

improvement has been verified by computational and experimental results [5,6]. The impedance match between the TE and the PTC should receive more attention also. Martin and Martin [7] used an empty volume as an impedance matching element to match properly the required large volumetric flow at the linear compressor with the smaller volumetric flow at the PTC, which was designed for operation at 106 K with a cooling power in excess of 2 kW driven by a 20 kW linear compressor. Conventionally, in order to decrease the cooling loss, a small scale Stirling type PTC is connected to a linear compressor or a thermoacoustic engine through a short tube with a small dead volume [8,9]. Luo et al. proposed the idea of an acoustic amplifier to connect a TE with a PTC. They pointed out that with an acoustic amplifier of appropriate length and diameter, the input pressure amplitude of the PTC can be increased. The acoustic amplifier adopted by Luo is a 2.7 m long tube

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with a diameter of 4 mm. With nitrogen as working gas, the pressure ratio at the blocked end of the acoustic amplifier is increased to 1.47, while it is 1.25 with a Stirling type TE [10]. The operating principle of an acoustic amplifier was presented, but the detailed transmission characteristics and some limitations were not investigated further.

In fact, an acoustic amplifier dissipates an amount of acoustic power when amplifying the pressure amplitude. It is important to find ways to improve its performance by suppressing the power consumption. Up to now, all the acoustic amplifiers are of uniform diameter, such as those used by Luo and Dai [10,11]. We call such an amplifier the single stage acoustic amplifier. However, when the diameter of the waveguide of a TE is much larger than that of the inlet tube of a PTC, an acoustic amplifier with uniform diameter cannot transfer enough acoustic power and leads to much dissipation.

In this paper, the transmission characteristics of the single stage acoustic amplifier are studied firstly. Secondly, the effect of output resistance impedance on the amplifying ability of an acoustic amplifier is investigated. Finally, the novel idea of cascade amplifier is proposed, which arranges tubes with different diameters in a series to increase further the amplifying ability and decrease the acoustic power dissipation. The experimental results in Section 4 verify the theoretical analysis in Section 2 and the computational results in Section 3 well.

2. Theoretical analysis

To be brief, an acoustic amplifier is a segment of tube with a designed length and diameter. According to linear thermoacoustic theory [12,13], the momentum and continuity equations of a short tube are as follows:

$$dp_1 = -\frac{i\omega\rho_m dx/A}{1-f_v} U_1 \quad (1)$$

$$dU_1 = -\frac{i\omega A dx}{\gamma p_m} [1 + (\gamma - 1)f_k] p_1 + \frac{(f_k - f_v)}{(1 - f_v)(1 - \sigma)} \frac{dT_m}{T_m} U_1 \quad (2)$$

where p_1 and U_1 are the pressure amplitude and volume flow rate amplitude, respectively, and ω is the angular frequency. ρ_m , T_m , p_m , γ and σ are mean density, mean temperature, mean working pressure, specific heat ratio and Prandtl number of the working fluid, respectively. f_v and f_k are the viscous and thermal distribution functions, respectively. A is the flow area of the tube. For simplicity, the effect of solids on the gas properties is neglected [14]. Since there is no remarkable temperature gradient along the amplifier, the second term on the right side of Eq. (2) is zero. The control equations are reduced as:

$$\frac{dp_1}{dx} = -ZU_1 \quad (3)$$

$$\frac{dU_1}{dx} = -Yp_1 \quad (4)$$

Eqs. (3) and (4) are two first order ordinary differential equations, where Z and Y are $\frac{i\omega\rho_m/A}{1-f_v}$ and $\frac{i\omega A}{\gamma p_m} [1 + (\gamma - 1)f_k]$, respectively. If the inlet boundary conditions are given as:

$$U_1(0) = U_{\text{lin}}, \quad p_1(0) = p_{\text{lin}}$$

The pressure and volume flow rate amplitudes at a given location x along the waveguide can be obtained by solving the above control equations as follows:

$$p_1 = \text{ch}(\sqrt{YZ}x) p_{\text{lin}} - \sqrt{\frac{Z}{Y}} \text{sh}(\sqrt{YZ}x) U_{\text{lin}} \quad (5)$$

$$U_1 = -\sqrt{\frac{Y}{Z}} \text{sh}(\sqrt{YZ}x) p_{\text{lin}} + \text{ch}(\sqrt{YZ}x) U_{\text{lin}} \quad (6)$$

where ch and sh denote the hyperbolic cosine and sine, respectively. The transfer matrix is written as:

$$\begin{bmatrix} p_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} \text{ch}(\sqrt{YZ}x) & -\sqrt{\frac{Z}{Y}} \text{sh}(\sqrt{YZ}x) \\ -\sqrt{\frac{Y}{Z}} \text{sh}(\sqrt{YZ}x) & \text{ch}(\sqrt{YZ}x) \end{bmatrix} \begin{bmatrix} p_{\text{lin}} \\ U_{\text{lin}} \end{bmatrix} \quad (7)$$

Then, for an acoustic amplifier with the length l , the amplifying ratio of the output pressure amplitude p_{out} to p_{lin} is deduced from Eq. (7) as:

$$\left| \frac{p_{\text{out}}}{p_{\text{lin}}} \right| = \left| \frac{1}{\text{ch}(\sqrt{YZ}l) + \frac{\sqrt{Z/Y}}{R_{\text{out}}} \text{sh}(\sqrt{YZ}l)} \right| \quad (8)$$

where R_{out} is the impedance at the output end of the amplifier. Obviously, the amplifying ratio is determined by the geometry of the tube, the properties of the working gas and the output impedance of the tube. When the output end is blocked, the output resistance impedance is infinite, and Eq. (8) is simplified to the expression in Ref. [10].

3. Computational results

The computation model shown above can be performed by using DeltaE (Design Environment for Low Amplitude ThermoAcoustic Engines) developed by Ward and Swift [14,15]. So, the computations in this paper are performed based on DeltaE.

3.1. Characteristics of single stage amplifier

Fig. 1 shows the effect of the length of a single stage acoustic amplifier on the output pressure amplitude and input acoustic power with nitrogen of 2.20 MPa as working gas. The amplifier is a stainless steel tube with the inner diameter of 29 mm. The output resistance impedance is infinite, and the inlet pressure amplitude is set at 0.05 MPa. As is shown, the output pressure amplitude increases monotonously until the length of the amplifier approaches 1/4 wavelength, about 3.86 m, and then begins to decrease. The maximal amplifying ratio is 9.5. When the

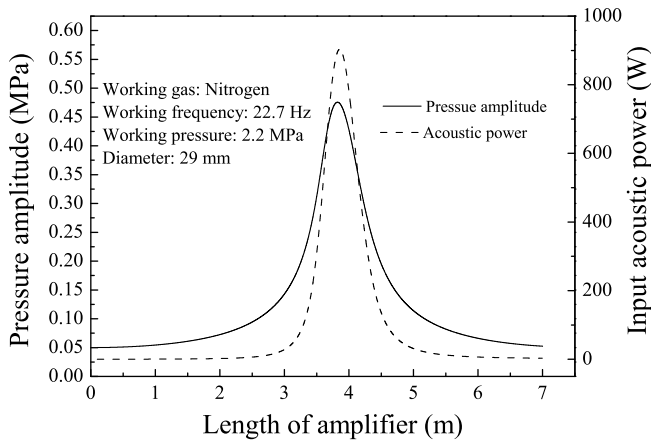


Fig. 1. Effect of length on output pressure amplitude and input acoustic power.

length of the amplifier exceeds one wavelength, other peaks of output pressure amplitude of lower amplifying ratio occur, which shows definite periodicity. It should be pointed out that nonlinear effects are enhanced with the increase of output pressure amplitude [12]. When the output pressure amplitude gets larger than 10% of the mean pressure, linear thermoacoustic theory cannot predict the output pressure amplitude accurately [10,16]. From the viewpoint of impedance, the acoustic amplifier is an acoustic load. It consumes an amount of acoustic power when amplifying the pressure amplitude. It is shown that the required input acoustic power increases with the output pressure amplitude. When the amplifying ratio is 4, the input acoustic power is about 85 W, while the input power reaches 903 W when the amplifying ratio is 9.5.

Fig. 2 shows the axial distribution of the pressure amplitude along single stage acoustic amplifiers with different lengths. The horizontal axis denotes the lengths of the amplifiers. The input pressure amplitude is also set at 0.05 MPa. As is shown, the length has a significant effect on the pressure amplitude distribution along an amplifier.

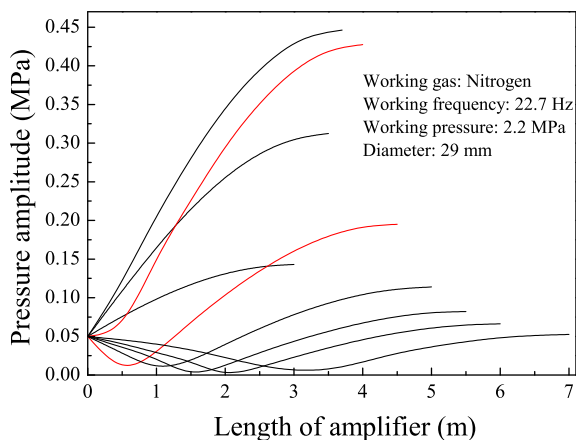


Fig. 2. Distribution of pressure amplitude along single stage acoustic amplifier.

The output pressure amplitude (at the blocked end of the amplifier) increases with the length of the amplifier monotonously when the amplifier is shorter than $1/4$ wavelength and begins to decrease when the length exceeds $1/4$ wavelength (about 3.86 m), which is consistent with Fig. 1. On the other hand, the pressure amplitude along the length of a particular amplifier increases monotonously when it is shorter than $1/4$ wavelength. When it gets longer, other pressure amplitude inflexions occur, which resemble the shapes of triangle cosine or sine functions. In spite of the length of the amplifier, a pressure amplitude peak always occurs at the blocked end.

3.2. Effect of output impedance on output pressure amplitude

Fig. 3 shows the effect of output resistance impedance on the output pressure amplitude of the single stage acoustic amplifier. The diameter of the amplifier and the working conditions are the same as in Section 3.1. As is shown, the output pressure amplitude has a strong dependence on the output resistance impedance. When the output resistance is $2.0 \times 10^7 \text{ N s/m}^5$, the amplifying ability of the acoustic amplifier is much weaker. When the output resistance is $7.0 \times 10^7 \text{ N s/m}^5$, the obtained maximal output pressure amplitude reaches 0.253 MPa and the corresponding amplifying ratio is about 5.1. In order to obtain a good match between a TE and a PTC by an acoustic amplifier, the impedance of the PTC should be deliberately designed.

3.3. Cascade acoustic amplifier

As shown in Fig. 1, when the output pressure amplitude is more than 0.15 MPa, the acoustic power consumption in the amplifier rises sharply, which is due to the near resonance working condition. Under such working condition, the velocity and pressure oscillations are almost in phase, and the volume flow rate gets high, which leads to large dissipation in the amplifier. By using a cascade method, this

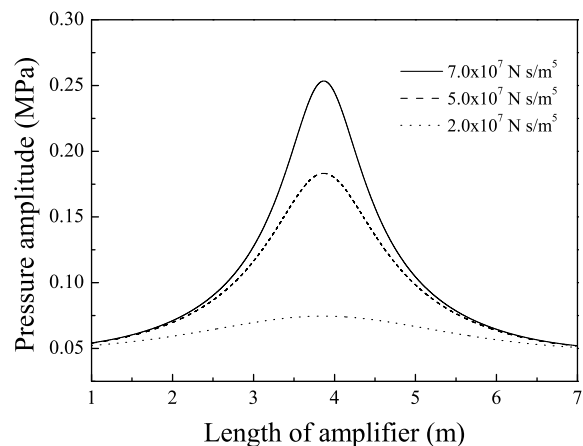


Fig. 3. Effect of output resistance impedance on output pressure amplitude.

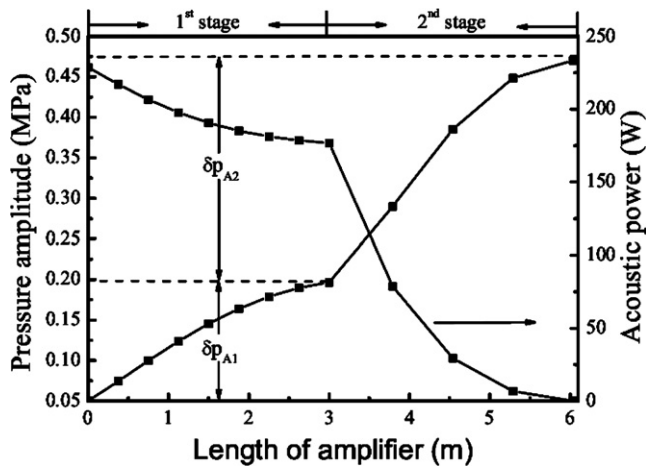


Fig. 4. Performance of a two-stage cascade acoustic amplifier. The amplifier is divided into 12 segments.

problem can be solved. Fig. 4 shows the performance of a two-stage cascade acoustic amplifier. The first stage is a 3 m long stainless steel tube with an inner diameter of 29 mm. The second stage is a 3 m long tube with an inner diameter of 8 mm. A short tapered tube (from the diameter of 29 mm to 8 mm) is set between the two stages. The working conditions are the same as in Section 3.1. As is shown, the cascade amplifier amplifies the pressure amplitude stage by stage. The inlet pressure amplitude of the second stage is the output amplitude of the first stage. The first stage amplifies the pressure amplitude from 0.05 MPa to about 0.20 MPa (denoted by δp_{A1}), and the second stage amplifies the pressure amplitude from 0.20 MPa to about 0.48 MPa (denoted by δp_{A2}). The amplifying ratio (4.0) of the first stage is much larger than that (2.86, as shown in Fig. 1) of a single stage amplifier of the same length and diameter, which is due to the influence of the second stage. The total amplifying ratio of the two-stage amplifier

is about that of a single stage amplifier under resonance working condition (as shown in Fig. 1) with much smaller acoustic power consumption. The input power of the two-stage amplifier is only 225.5 W, while the input power of the single stage amplifier reaches 903 W for the resonance working condition. Furthermore, the theoretical maximal amplifying ratio of a single stage amplifier is about 9.5 (as shown in Fig. 1), while the cascade acoustic amplifier can break this limitation by lengthening either stage of the cascade amplifier a little. As shown by the acoustic power distribution curve, the acoustic power consumption in the second stage is much larger than that in the first stage due to the much higher velocity.

4. Experimental verification

In order to verify the simulation, a number of experiments were performed. The amplifiers studied are connected to a thermoacoustic Stirling engine [17]. In the measurements, the thermoacoustic engine only acts as a pressure source, producing the pressure oscillation and transmitting it to the amplifiers. The engine and acoustic amplifiers are shown in Fig. 5 schematically. As shown, the amplifiers are connected at the compliance of the engine. The inner diameter of the compliance of the engine is 100 mm. The single stage amplifier is a stainless steel tube with variable length (with the inner diameter of 29 mm). The two-stage cascade amplifier is composed of three parts. The first stage is a stainless steel tube of the same diameter as the single stage, and the second stage is a copper tube with the inner diameter of 8 mm. A short tapered brass tube and a ball valve are set between the first and second stage. For convenience, the tapered tube and ball valve are not removed in the experiments on the single stage amplifier, which are taken into account in the computational model. In order to keep the gas properties constant, the amplifiers are cooled by water.

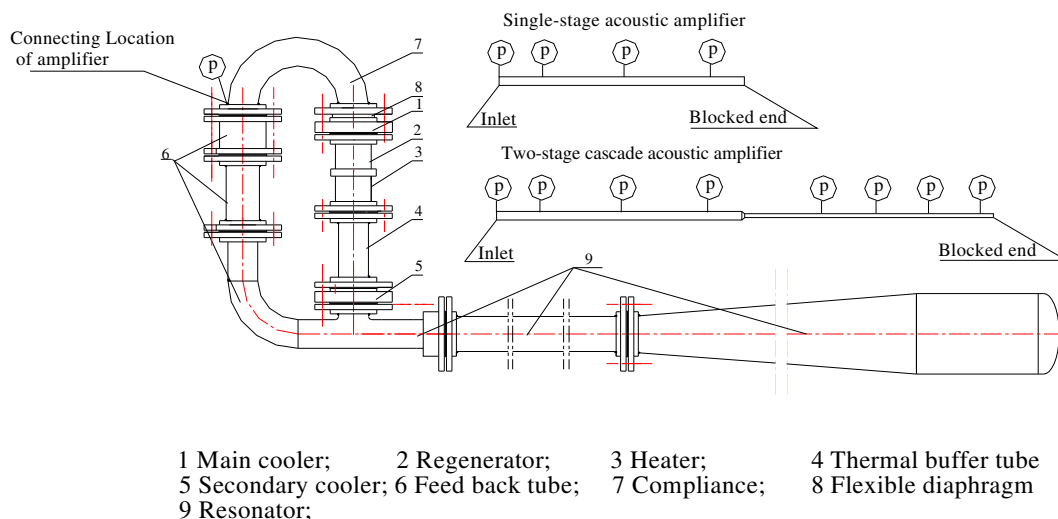


Fig. 5. Schematic diagram of thermoacoustic engine and acoustic amplifiers. Symbol P shows the arrangement of pressure sensors.

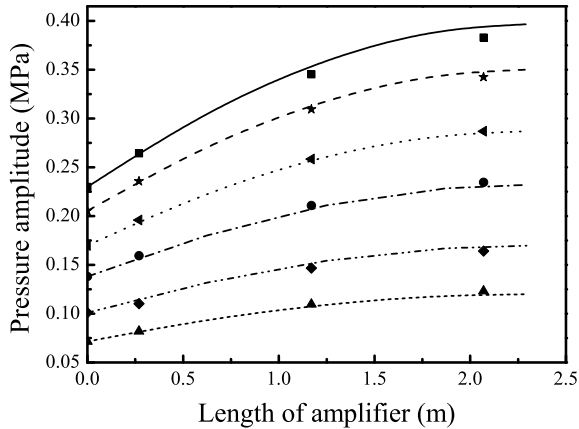


Fig. 6. Pressure amplitude distribution along a single stage acoustic amplifier. Curves are computational results and scattered points are experimental results. The exact locations of the pressure sensors along the amplifier can be obtained by the abscissa values of the scattered experimental points.

The pressure data acquisition system is comprised of pressure sensors, data acquisition clip, computer and a self developed program based on Labview 6.1 by National Instruments Inc. (NI) [18]. The symbol P in Fig. 5 shows the locations of the pressure sensors. The pressure sensors are of linear silicon piezoelectric type and were supplied by Infineon Technologies, Germany. The pressure sensors were calibrated before the measurements, and the typical linearity error is $\pm 0.15\%$ of the readings. The data acquisition clip is NI PC-1200, 12 bits in precision and 10 kS/s in sampling frequency.

Fig. 6 shows the pressure amplitude distribution along the single stage acoustic amplifier with different inlet pressure amplitudes. The amplifier is a 2.29 m long stainless steel tube with the inner diameter of 29 mm. A 35 mm long tapered brass tube (from the diameter of 29 mm to 8 mm) and a ball valve are set at the output end. The working pressure and frequency are about 2.20 MPa and 22.70 Hz, respectively. Because of the variation of heating temperature and the heat generated by viscous dissipation in the experimental processes, the working pressure and frequency change a little, which are also taken into account in the computations. The inlet pressure amplitudes in the computations and experiments are set the same. As is shown, the measured inlet pressure amplitude ranges from

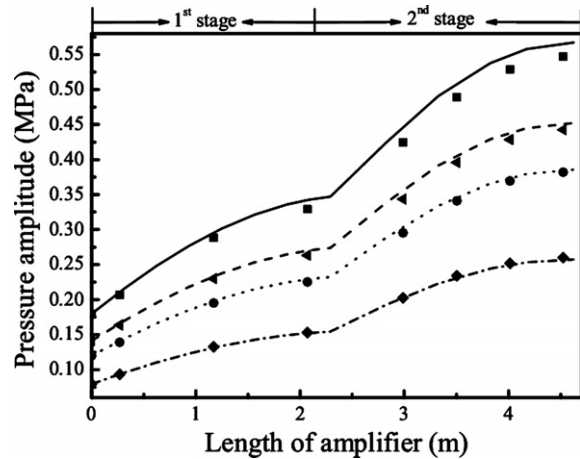


Fig. 7. Pressure amplitude distribution along a two-stage acoustic amplifier. Curves are computational results and scattered points are experimental results. The exact locations of the pressure sensors along the amplifier can be obtained by the abscissa values of the scattered experimental points.

0.0563 MPa to 0.23 MPa. The computational and experimental amplifying ratio ranges from 1.66 to 1.74. The amplifying ratio decreases with the rise of inlet pressure amplitude. The computational results are in relatively good agreement with the experimental results, especially when the inlet pressure amplitudes are below 0.18 MPa. When the pressure amplitude gets larger, the deviation between computational and experimental results is mostly caused by nonlinear effects, which are not considered in linear thermoacoustic theory.

Fig. 7 shows the pressure amplitude distribution along the two-stage acoustic amplifier. The first stage is a 2.29 m long stainless steel tube with the inner diameter of 29 mm. The second stage is a 2.30 m long copper tube with the inner diameter of 8 mm. A 35 mm long tapered brass tube is set between the two stages. The inlet pressure amplitudes in the experiments and computations are set the same. The working condition is the same as above. The distribution curves and scattered points show the cascade amplification of pressure amplitude clearly. When the inlet pressure amplitude is less than 0.15 MPa, the computational results are in good agreement with the experimental results. Compared to Fig. 6, the two-stage acoustic amplifier shows much higher amplifying ability. Influenced by the second

Table 1

Performance comparison between a single stage and a two-stage cascade acoustic amplifier

	Single stage		Two stage	
	Experiment	Computation	Experiment	Computation
P_{Ai} (MPa)		0.1804		0.1797
P_{A1} (MPa)	0.3037	0.3055	0.3293	0.3472
P_{A2} (MPa)			0.5470	0.5671

Note: Due to the lengths of the connecting tubes, flanges and valve, the experimental pressure amplitudes shown in the table are not exactly at the end of each amplifier as in the computations. P_{Ai} denotes the inlet pressure amplitude, P_{A1} denotes the output pressure amplitude of the single stage acoustic amplifier and the first stage of the two-stage acoustic amplifier, and P_{A2} denotes the output pressure amplitude of the second stage of the two-stage acoustic amplifier.

stage, the amplifying ability of the first stage is significantly enhanced. The detailed comparison is shown in Table 1. Although the inlet pressure amplitude of the two-stage amplifier is a little less than that of the single stage amplifier, the output pressure amplitude of the first stage is much larger than that of the single stage amplifier with the same length, which has been predicted in Section 3.3. The two-stage cascade amplifier shows much higher amplifying ability. In the experiment, it amplifies the inlet pressure amplitude from 0.1797 MPa (corresponding to the pressure ratio of 1.177) to the output pressure amplitude of 0.547 MPa. The corresponding pressure ratio is 1.62, which is the highest pressure ratio obtained by a TE to date.

5. Conclusions

An acoustic amplifier is powerful to amplify pressure amplitude. Theoretically, the amplifying ability of a single stage acoustic amplifier is maximal when its length approaches $1/4$ wavelength. It is shown that the pressure amplitude distribution along an amplifier possesses the same shape as a sine or cosine function, and a pressure amplitude peak always occurs at the blocked end of the amplifier. The output resistance impedance has a strong effect on the output pressure amplitude of a single stage acoustic amplifier. Furthermore, the acoustic power dissipation increases with amplification ratio.

In order to increase the amplification ratio and reduce the acoustic power consumption, a novel cascade acoustic amplifier is proposed. As is proved by the computations and experiments, the cascade acoustic amplifier has a much higher amplifying ability with reduced acoustic power consumption.

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